Toward Microscopic Equations of State for Core-Collapse Supernovae from Chiral Effective Field Theory

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Motivation

- Chiral effective field theory is an approximation to Quantum Chromodynamics
  - Provides a modern framework for understanding the structure and dynamics of nuclear many-body systems
- Our aim is to extend the application of chiral effective field theory to describe the nuclear equation of state required for supercomputer simulations of core collapse supernovae
  - Large computational resources on the order of one million CPU hours are needed
- We investigate the use of graphics processing unit (GPUs)
  - Reduces the computational cost significantly
  - Enables more accurate and precise description of the nuclear equation of state

Core-Collapse Supernovae

- Stars between $10\ - 30\ M_{\odot}$ reach sufficiently high temperatures to fuse nuclei up to iron
  - When the iron core reaches $1.4\ M_{\odot}$, electron degeneracy pressure can no longer support the iron core against gravitational collapse
- Short-range nuclear forces acting over femtoscale distances eventually halt the collapse
  - A shockwave rebounds against the inward-falling matter above
  - Electron capture in the core releases abundant neutrinos that deposit additional energy behand the shockwave

$$\boldsymbol{e + p\ \rightarrow n + v_e}$$

  - This potentially leads to a successful supernova explosion
- A neutron star or a black hole may be left over after the supernova

Core-Collapse Supernovae (cont.)

The nuclear equation of state plays a critical role in simulating stellar core collapse and the hydrodynamic evolution of the resulting supernova explosion

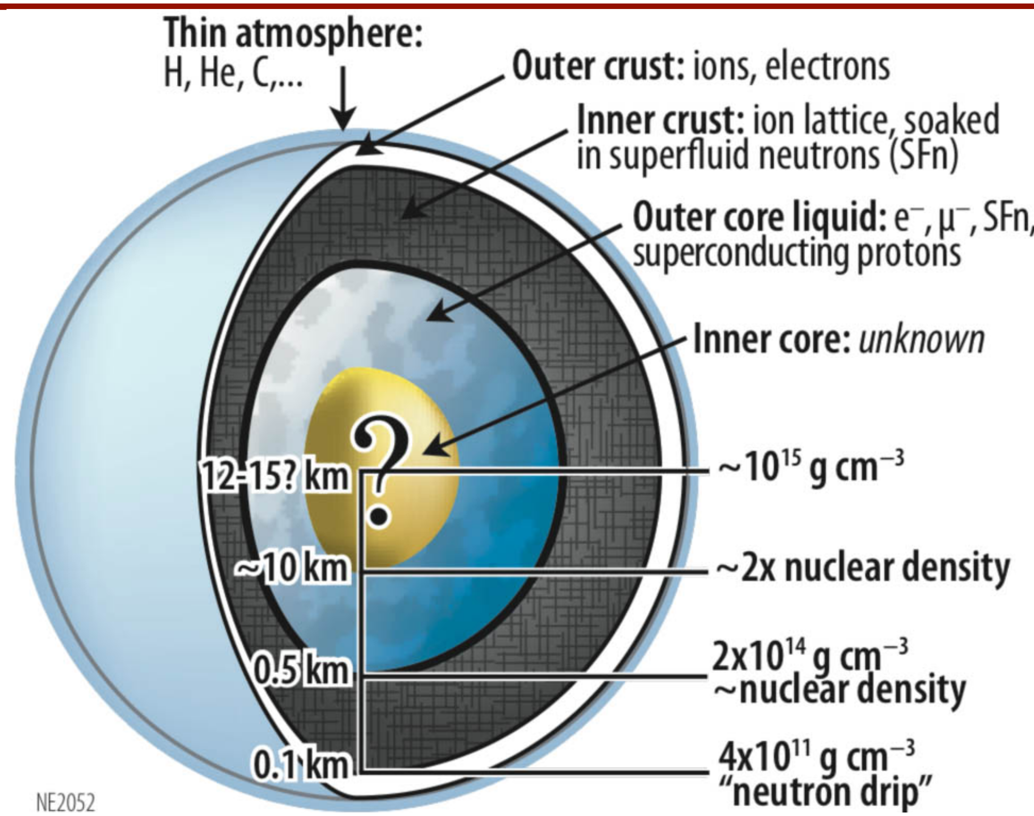


Figure 1: Neutron Star Diagram

Hot and Dense Matter Equation of State

Realistic two- and three-body chiral nuclear forces are used to calculate the free energy of nuclear matter at varying temperature, density, and composition. The free energy per nucleon in infinite homogeneous nuclear matter can be expanded in perturbation theory as follows:

$$\bar{F}(T,\rho,\delta) = \bar{F}_o(T,\rho,\delta) + \lambda\bar{F}_1(T,\rho,\delta) + \lambda^2\bar{F}_2(T,\rho,\delta) + \lambda^3\bar{F}_3(T,\rho,\delta) + \mathcal{O}(\lambda^4)$$

Where,

T − temperature
ρ − total nucleon density
δ − isospin-asymmetry parameter

Hot and Dense Matter Equation of State (cont.)

$$\rho E^{(1)} = \frac{1}{2}\sum_{12} n_1 n_2 \langle 12|(\bar{V}_{NN} + \bar{V}_{NN}^{med}/3)|12\rangle$$

$$\rho E^{(2)} = -\frac{1}{4}\sum_{1234} \left|\langle 12|\bar{V}_{eff}|34\rangle\right|^2 \frac{n_1 n_2 \bar{n}_3 \bar{n}_4}{e_3 + e_4 - e_1 - e_2}$$

$$\rho E^{(3)}_{pp} = \frac{1}{8}\sum_{123456} \langle 12|\bar{V}_{eff}|34\rangle \langle 34|\bar{V}_{eff}|56\rangle\langle 56|\bar{V}_{eff}|12\rangle \times \frac{n_1 n_2 \bar{n}_3 \bar{n}_4 \bar{n}_5 \bar{n}_6}{(e_3 + e_4 - e_1 - e_2)(e_5 + e_6 - e_1 - e_2)}$$

$$\rho E^{(3)}_{hh} = \frac{1}{8}\sum_{123456} \langle 12|\bar{V}_{eff}|34\rangle \langle 34|\bar{V}_{eff}|56\rangle\langle 56|\bar{V}_{eff}|12\rangle \times \frac{\bar{n}_1 \bar{n}_2 n_3 n_4 n_5 n_6}{(e_1 + e_2 - e_3 - e_4)(e_1 + e_2 - e_5 - e_6)}$$

$$\rho E^{(3)}_{ph} = -\sum_{123456} \langle 12|\bar{V}_{eff}|34\rangle \langle 54|\bar{V}_{eff}|16\rangle\langle 36|\bar{V}_{eff}|52\rangle \times \frac{n_1 n_2 \bar{n}_3 \bar{n}_4 n_5 \bar{n}_6}{(e_3 + e_4 - e_1 - e_2)(e_3 + e_6 - e_2 - e_5)}$$

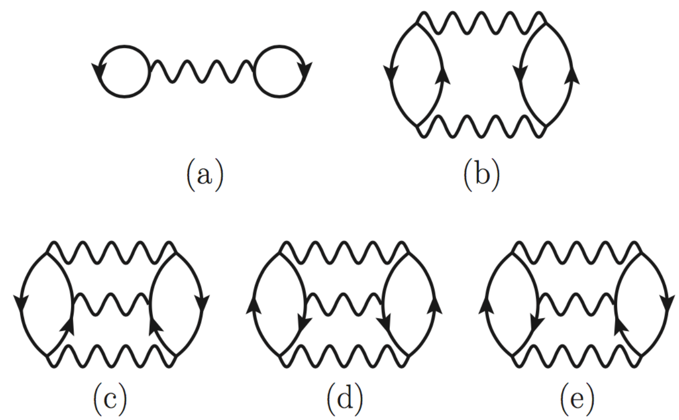


Figure 2: First-, second-, and third- order diagrammatic contributions to the ground state energy density of nuclear matter by Holt et. al.[1] The wavy lines indicate the (antisymmetrized) density-dependent NN interaction derived from chiral two- and three-body forces.

Hot and Dense Matter Equation of State (cont.)

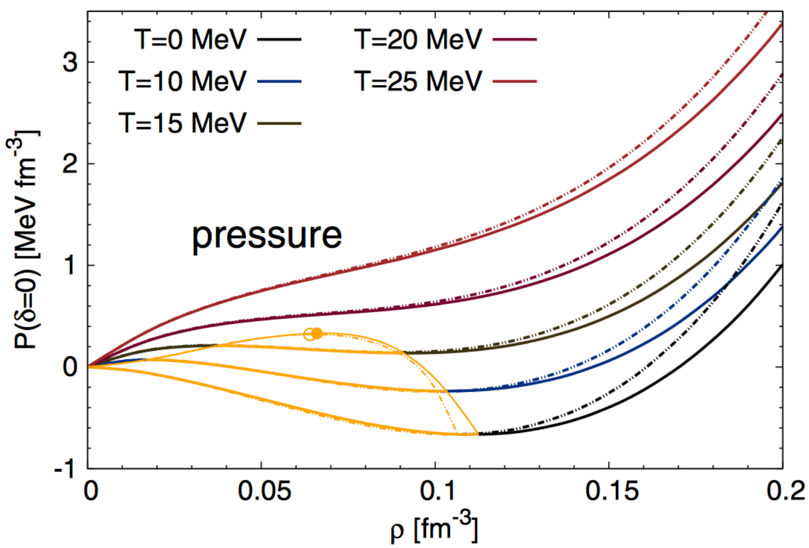


Figure 3: Pressure in isospin symmetric nuclear matter for temperatures in the range $T = 0 - 25MeV$ by Wellenhofer et. al.[2]

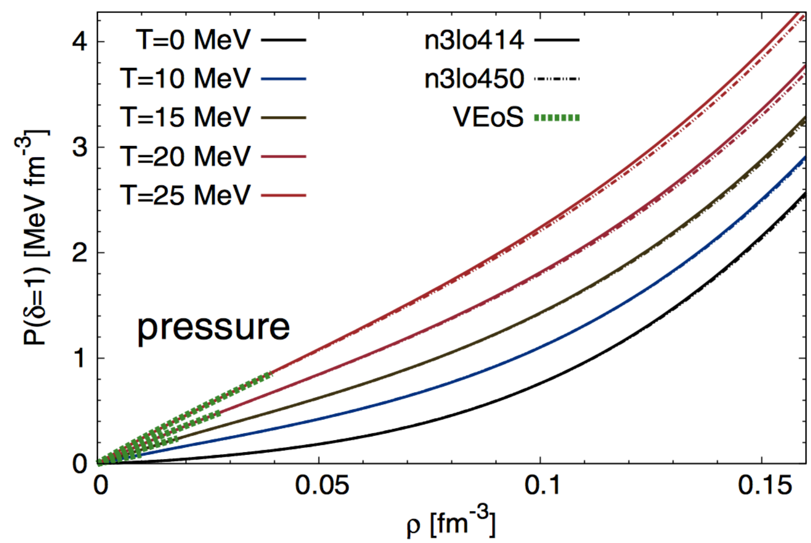


Figure 4: Pressure in pure neutron matter by Wellenhofer et. al.[2]

Numerical Optimization on GPUs

- We explore the possibility to calculate the nuclear equation of state using massive parallel computing on GPU accelerators

- CUDA:
    - A parallel computing platform invented by NVIDIA
    - Allows for general use GPU
    - Provides a significant increase in processing power

Figure 5: CUDA Logo

CUDA Memory Structure

- Kernel:
  - A function called by the host (CPU) that runs on the device (GPU).
  - It allows for massive parallelism

- Blocks containing threads are dedicated to do the specified computational work.
  - The GPU architecture allows for thousands of independent threads to execute simultaneously.
  - May lead to significantly improved computational efficiency

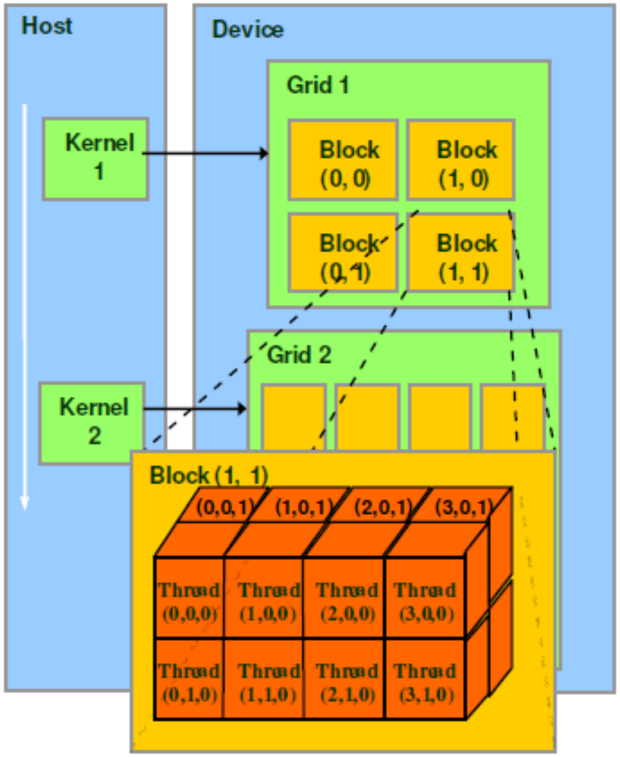
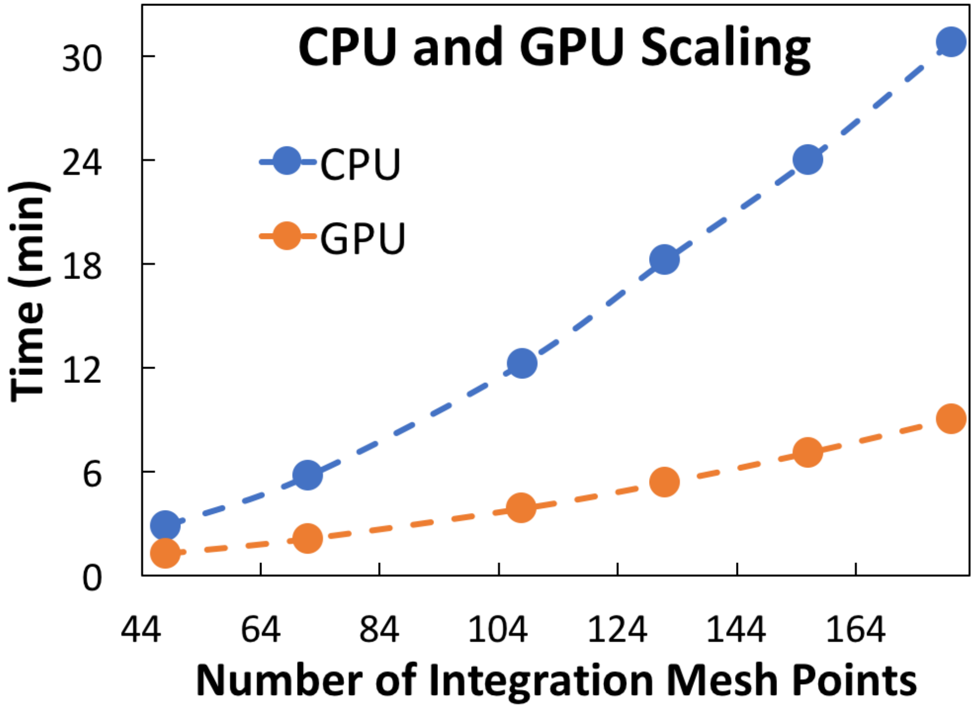


Figure 6: CUDA Memory Model

Results



Mesh Points	Serial (min:sec)	Parallel (min:sec)	Speedup
48	2:51	1:16	2.244
72	5:45	2:08	2.696
108	12:15	3:52	3.168
132	18:14	5:21	3.408
156	24:01	7:04	3.397
180	30.49	9:01	3.424

Figure 6: The plot to the left compares GPU and CPU runtime as a function of desired numerical precision (encoded in the number of integration mesh points). The GPU code exhibits a much more favorable scaling compared to the optimized CPU version.

Conclusions:

- The current GPU implementation offers more efficient algorithm than the original CPU program, especially the highest numerical precisions achieved.
- This provides a first step toward faster calculations needed for constructing equation of state tables for astrophysical simulations.

Future work:

- Optimization of the current GPU algorithm
    - Speed up of 5 – 10 times would be more desirable

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References

1. J. W. Holt and N. Kaiser, "Equation of State of Nuclear and Neutron Matter at Third-Order in Perturbation Theory from Chiral Effective Field Theory", Physical Review C **95**, 034326 (2017).

2. C. Wallenhofer, J. W. Holt, and N. Kaiser, "Thermodynamics of Isospin-Asymmetric Nuclear Matter form Chiral Effective Field Theory", Physical Review C **92**, 015801 (2015).

Figures:

- Figure 1: https://heasarc.gsfc.nasa.gov/docs/nicer/nicer_about.htm
- Figure 5: http://www.nvidia.com/object/io_1221568471314.html
- Figure 6: https://sites.google.com/site/cudapros/website-builder